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Optimal strategy for an integrated system with variable production rate when the freight rate and trade credit are both linked to the order quantity

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ABSTRACT

This paper presents an integrated inventory model with variable production rate and price-sensitive demand rate. The buyer's purchases trade credit linked to the order quantity offered by the supplier. In addition, the buyer pays the freight charge according to a weight schedule. This paper attempts to offer a best policy that aims at maximizing the joint total profit while the trade credit and freight rate are simultaneously linked to the order quantity. The same policy also incorporates considerations on the optimal retail price, order quantity and delivery decision. We provide possible solutions for the buyer and the supplier to collaboratively agree on inventory control, warehouse management, transportation logistics, delivery and billing. Our study demonstrates that significant profit increase for the entire supply chain can be achieved by linking both trade credit policy and freight rate policy to order quantities. An algorithm is furnished to determine the optimal solution. In addition, numerical examples and sensitivity analysis are presented to illustrate the theoretical results.

1. Introduction

To promote sales, suppliers often award trade credit to buyers. As shown in previous literature, such as Petersen and Rajan (1995, 1997), Nielsen (2002) and Fisman and Love (2003), buyers may be financed by their suppliers rather than by financial institutions. Since Goyal (1985) first established an economic order quantity (EOQ) model that allows for delayed payment terms or credit terms, many researchers have addressed this topic with interesting results: e.g. Kim et al. (1995), Aggarwal and Jaggi (1995), Brigham (1995), Jamal et al. (1997), Arcelus and Srinivasan (2001), Teng (2002), Biskup et al. (2003), Salameh et al. (2003), Teng et al. (2005, 2006), Chung and Huang (2007), Huang and Hsu (2008) and Liao (2008). All previous models implicitly assumed that credit terms are independent of the order quantity. In reality, suppliers may offer favorable credit terms to encourage buyers to order larger quantities. Khouja and Mehrez (1996) were the first proponents to discuss supplier credit policies in the EOQ model where credit terms are linked to the order quantity. Later, Chang et al. (2003) established an EOQ model for deteriorating items, in which the supplier credit is linked to order size. Other researchers dealt with similar order size and trade credit relations such as Shinn and Hwang (2003), Chung and Liao (2004, 2006) and Chung et al. (2005).

Given the intense market competition of late, transportation costs are critical in many operating decisions. An appropriate transportation cost function should be

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incorporated into models with all other relevant costs. Tersine et al. (1989) published an economic inventory-transport model with freight discounts. The models proposed by Lee (1986), Hwang et al. (1990) and Tersine and Barman (1991, 1994) combined freight discounts with all-unit or incremental discounts. Russell and Krajewski (1991) presented an analytical approach to find the optimal order quantity that minimized the total purchase cost, which reflects both transportation economies and quantity discounts. Shinn et al. (1996) allowed freight discounts to be linked to credit terms, when formulating the retailer's optimal purchase lot size model. Swenseth and Godfrey (2002) proposed a model in which two freight rate functions, the inverse function and the adjusted inverse function, were incorporated into the total annual logistics cost function to determine the impact on purchasing decisions. Lately, Abad and Aggarwal (2005) presented a model that allows for over-declaring the shipment weight and for less-than-truckload (LTL) or truckload (TL) shipments.

In the existing literature on classical inventory models, the production rate and the unit production cost are assumed to be constant. In reality, it has been observed that the production rate and the unit production cost vary with changes in the demand rate in many situations. Cheng (1991) proposed an EOQ model to shed light on the relationship between demand-dependent unit production cost and imperfect production processes. Khouja and Mehrez (1994) extended the economic production lot size model to cases where the production rate is a decision variable. In their model the unit production cost is treated as a function of the production rate. Bhunia and Maiti (1997) presented two inventory models in which the production rate depended upon on-hand inventory for the first model and upon demand for the second. Manna and Chaudhuri (2001) discussed an EOQ model with deteriorating items in which the production rate is related to the time-dependent demand rate.

Previous studies about trade credit and transportation cost were concerned only with decisions that minimized the cost for the buyer or maximized the profit for the supplier. However, the complicated interaction and co-operation opportunities were considered trivial and ignored in their consideration. In reality, independent and conflicting production plans were often made by members in the supply chain to achieve individual goals. Therefore, an aggregate approach to planning with an emphasis on improving supply chain efficiency is needed to help business survive in this fiercely competitive world. Goyal (1976) first developed an integrated inventory model focusing on a single supplier–single buyer scenario. Afterwards, Banerjee (1986) further extended Goyal's model with one additional assumption that the suppliers follow a lot-for-lot shipment policy. Many researchers (e.g., Joglekar, 1988; Lu, 1995; Goyal, 1995; Viswanathan, 1998; Hill, 1999; Kelle et al., 2003, 2007) followed the same line of reasoning but proposed more batching and shipping policies for an integrated inventory model. These researches on the integrated supplier–buyer inventory study focused on the production–distribution schedule in terms of the number and size of the lots transferred between both parties. Recently, some researchers also considered a pricing and lot-size policy in an integrated inventory model that includes trade credit considerations (e.g., Abad and Jaggi, 2003; Ouyang et al., 2005; Luo 2007). However, the effects that transportation charges may have on policy alternatives were ignored in their study.

Cost reduction from either a trade credit or a discount in freight rate gives the buyer incentive to lower the retail price in order to increase market share. Change in price affects demand, which in turn affects decisions on production, shipping, inventory, and trade credit policies. In this paper we will consider a scenario in which a single supplier and a single buyer deal with a price-sensitive product. The production cost is assumed to be a convex function of the production rate. We would like to explore the process of arriving at the best policy that includes the optimal retail price, order quantity and delivery, to maximize the joint total profit in a model where both trade credit and freight rate are linked to order quantity. An algorithm is provided to determine the optimal solution. Numerical examples are presented to demonstrate the results of the proposed model. Furthermore, a sensitivity analysis is done on some relevant parameters in the optimal solutions and the result is included.

2. Assumptions and notations

The proposed model is developed based on the following assumptions and notations:

1. There is a single supplier and a single buyer for a single product in this paper.
2. Shortage considerations are excluded.
3. The carrying cost rate is r_V for the supplier and r_B for the buyer; the interest charge is not considered.
4. The market demand for the product is assumed to be sensitive to the buyer's selling price p and is defined as $D(p) = ap^{-\delta}$, where a is a scaling factor that is greater than 0; δ is a price-elasticity coefficient that is greater than 1. For notational simplicity, $D(p)$ and D will be used interchangeably in this paper.
5. The capacity utilization, ρ, is the ratio of the demand rate, D, to the production rate, R; it is always less than 1. ($\rho = D/R$ and $\rho < 1$.)
6. The same assumption used in Khouja (1995) is used here, the unit production cost $c(R)$ is a convex function of the production rate, R. That is, $c(R) = c_0 + c_1/R + c_2R$, where c_0, c_1 and c_2 are non-negative real numbers to be set to best fit the estimated unit production cost function. The fixed cost c_0 can be regarded as the material cost. The cost component c_1/R decreases as the production rate (R) increases, representing costs such as labor cost and energy cost, both are equally distributed over a large number of units. The third term (c_2R) denotes a cost component that increases with the production rate. Such cost would include additional tool or die wear at high production rate. For notational simplicity, $c(R)$ and c are used interchangeably in this paper.

7. For each unit of product, the supplier spends \$$c$ in production and receives \$$v$ from the buyer. The buyer then sells the product at \$$p$ to his/her customers. Here, p is greater than v, and v, in turn, is greater than c ($p>v>c$).
8. The supplier offers a credit period N_η, which is linked to order size in the schedule as follows:

η	Q	N_η
1	$\vartheta_1 \leqslant Q < \vartheta_2$	N_1
2	$\vartheta_2 \leqslant Q < \vartheta_3$	N_2
⋮	⋮	⋮
λ	$\vartheta_\lambda \leqslant Q < \vartheta_{\lambda+1}$	N_λ

where $0=\vartheta_1<\vartheta_2<\cdots<\vartheta_\lambda<\vartheta_{\lambda+1}=\infty$, each represents a boundary quantity. N_η denotes the credit period applicable to orders whose lot size Q falls in the interval ϑ_η to $\vartheta_{\eta+1}$ with $0<N_1<N_2<\cdots<N_\lambda$.
9. During the credit period, the buyer sells the items and uses the sales revenue to earn interest at a rate of I_{Be}. At the end of the credit period, the buyer pays the purchasing cost to the supplier and incurs a capital opportunity cost at a rate of I_{Bp} for the items remaining in stock. The buyer's interest earned per unit time is more than his/her capital opportunity cost per unit time from trade credit, i.e., $pI_{\mathrm{Be}}>vI_{\mathrm{Bp}}$.
10. In offering trade credit to the buyer, the supplier endures a capital opportunity cost at rate I_{Vp} during the period between product shipped and paid for, where $I_{\mathrm{Vp}} \leqslant I_{\mathrm{Bp}}$.
11. The buyer's replenishment cycle length is T and order quantity is Q $(=DT)$ per order. For each order, the buyer incurs an ordering cost, S_{B}.
12. During the production period, the supplier manufactures in batches of size nQ, where n is an integer, and incurs a batch setup cost S_{V}. Once the first Q units are produced, the supplier delivers them to the buyer and then continues making the delivery on average every $T(=Q/D)$ units of time until the supplier's inventory level falls to zero.
13. The supplier charges freight for shipping according to a weight schedule that is defined below:

ε	W	ψ_ε
1	$\mu_1 \leqslant W < \mu_2$	ψ_1
2	$\mu_2 \leqslant W < \mu_3$	ψ_2
⋮	⋮	⋮
M	$\mu_m \leqslant W < \mu_{m+1}$	ψ_m

where $0=\mu_1<\mu_2<\cdots<\mu_m<\mu_{m+1}=\infty$ is the boundary values of the freight weights at which freight rate-break occurs. ψ_ε denotes that the freight rate applicable to the shipping weight W falls in the interval μ_ε to $\mu_{\varepsilon+1}$ with $\psi_1>\psi_2>\cdots>\psi_m>0$. The shipping weight of the product is θ lbs per unit, i.e. $W=\theta Q$. The unit shipping cost is therefore the unit shipping weight times the freight rate, which is $F_\varepsilon=\theta\psi_\varepsilon$.

3. Model formulation

The buyer is presented with an order-size-based credit terms schedule and a freight rate discounts schedule. The approach adopted in Tersine and Barman (1991) is used here; we combine the two discount schedules in assumptions 8 and 13 into a restructured new discount schedule. The reorganized process is accomplished by identifying all possible lot-size intervals with no breaks within each interval. The restructured discount schedule becomes

J	W	M_j	F_j
1	$q_1 \leqslant Q < q_2$	M_1	F_1
2	$q_2 \leqslant Q < q_3$	M_2	F_2
⋮	⋮	⋮	⋮
K	$q_k \leqslant Q < q_{k+1}$	M_k	F_k

where $k \leqslant \lambda+m$. For $q_j \leqslant Q < q_{j+1}$, $j=1,2,\ldots,k$, the length of credit period offered by the supplier is M_j and the freight rate charged by the supplier is F_j dollars per unit, where $0=q_1<q_2<\cdots<q_k<q_{k+1}=\infty$, $0<M_1<M_2<\cdots<M_k$ and $F_1>F_2>\cdots>F_k>0$.

3.1. Buyer's total profit per unit time

For each order quantity $Q\in[q_j,q_{j+1})$, the buyer pays vQ to the supplier and receives pQ from the customers. Therefore, sales revenue per unit time is $(p-v)Q/T=(p-v)ap^{-\delta}$ for the buyer. In addition, the buyer incurs an ordering cost per unit time S_{B}/T, a transportation cost per unit time $F_jQ/T=ap^{-\delta}F_j$ and an inventory holding cost (excluding interest charges) per unit time $vr_{\mathrm{B}}Q/2=vr_{\mathrm{B}}ap^{-\delta}T/2$.

During the credit period, the buyer sells the item and uses the sales revenue to earn interest at a rate of I_{Be}. At the end of the permissible delay period, the buyer pays the purchasing cost to the supplier and incurs a capital opportunity cost at a rate of I_{Bp} for the items left unsold. For possible values of M_j and T, the buyer has the following two possible cases: (i) $T<M_j$, (ii)$T \geqslant M_j$. For details, see Fig. 1.

Case 1: $T<M_j$

In this case, the inventory is completely depleted before the delayed payment due date, so the buyer pays no opportunity cost for the items kept in stock. At the same time, the buyer uses the sales revenue to earn

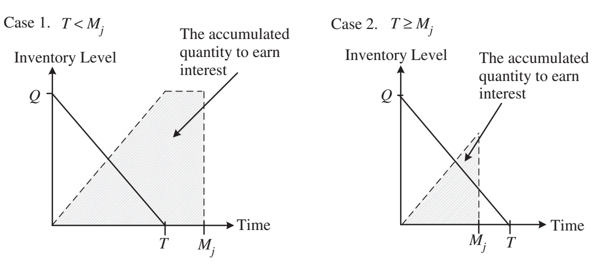


Fig. 1. The inventory level and accumulated interest earned for the buyer.

interest at a rate of I_{Be}. Therefore, the interest earned per unit time is

$$\frac{pI_{Be}}{T}\left[\int_0^T Dt\,dt + DT(M_j - T)\right] = ap^{-\delta+1}I_{Be}(M_j - T/2).$$

Case 2: $T \geqslant M_j$

This situation represents that the inventory is depleted either on the delayed payment due date or after. Since after the due date M_j the buyer still has some inventory, the capital opportunity cost per unit time is

$$\frac{vI_{Bp}}{T}\int_{M_j}^T D(T-t)\,dt = vI_{Bp}ap^{-\delta}(T-M_j)^2/(2T).$$

In either case, as the buyer does not pay the supplier until the end of the credit period, the buyer can use the sales revenue during the credit period to earn interest at a rate of I_{Be}. Therefore, the interest earned per unit time is

$$\frac{pI_{Be}}{T}\int_0^{M_j} Dt\,dt = (ap^{-\delta+1}I_{Be}M_j^2)/(2T).$$

Note that many researchers used different ways to calculate the interest earned and opportunity cost. In this paper, we use the Teng (2002) approach throughout this paper.

The total profit per unit time for the buyer is the total sales revenue minus the total relevant cost (which includes ordering cost, transportation cost, holding cost excluding interest charges and opportunity cost) plus interest earned. Therefore, the buyer's total profit per unit time can be expressed as

$$\mathrm{TBP}_j(p,T) = \begin{cases} \mathrm{TBP}_{1j}(p,T) & \text{if } T < M_j, \\ \mathrm{TBP}_{2j}(p,T) & \text{if } T \geqslant M_j, \end{cases} \quad (1)$$

where

$$\mathrm{TBP}_{1j}(p,T) = ap^{-\delta}(p-v) - \left[\frac{S_B}{T} + ap^{-\delta}F_j + \frac{vr_B ap^{-\delta}T}{2}\right] + ap^{-\delta+1}I_{Be}\left(M_j - \frac{T}{2}\right), \quad (2)$$

and

$$\mathrm{TBP}_{2j}(p,T) = ap^{-\delta}(p-v) - \left[\frac{S_B}{T} + ap^{-\delta}F_j + \frac{vr_B ap^{-\delta}T}{2} + \frac{vI_{Bp}ap^{-\delta}(T-M_j)^2}{2T}\right] + \frac{ap^{-\delta+1}I_{Be}M_j^2}{2T}. \quad (3)$$

3.2. *Supplier's total profit per unit time*

Throughout each production run, the supplier manufactures in batches of size nQ and incurs a batch setup cost S_V. The cycle length for the supplier is $nQ/D(=nT)$. Therefore, the supplier's setup cost per unit time is S_V/nT. The inventory carrying cost includes storage and handling expenses, insurance and taxes as well as the time value of capital tied up in inventories. With the unit production cost c, the carrying cost rate excluding interest charges r_V and the capital opportunity cost per dollar per unit time I_{Vp}, using the same approach as in Joglekar (1988), the supplier's carrying cost per unit time is given by

$$c(r_V + I_{Vp})\frac{ap^{-\delta}T}{2}[(n-1)(1-\rho)+\rho].$$

By offering credit terms to the buyer, the supplier gives up an immediate cash inflow until a later date. We use M_j to represent the time delay. With a finance rate I_{Vp}, the lost-opportunity cost per unit time for offering trade credit is $vI_{Vp}QM_j/T = vI_{Vp}ap^{-\delta}M_j$.

The total profit per unit time for the supplier is derived by taking out the total relevant cost (including the setup cost, inventory holding cost, and opportunity cost for the trade credit offered) from the total sales revenue. It is represented in the following function:

$$\mathrm{TVP}_j(n,T) = (v-c)ap^{-\delta} - \frac{S_V}{nT} - \frac{c(r_V+I_{Vp})ap^{-\delta}T}{2} \times[(n-1)(1-\rho)+\rho] - vI_{Vp}ap^{-\delta}Mj. \quad (4)$$

3.3. *The joint total profit per unit time*

Once the supplier and buyer have established a long-term strategic partnership by entering into a contract relationship, they can jointly determine the best policy for the entire supply chain system. Under this circumstance the joint total profit per unit time for the supplier and buyer is

$$\Pi_j(n,p,T) = \begin{cases} \Pi_{1j}(n,p,T) & \text{if } T < M_j, \\ \Pi_{2j}(n,p,T) & \text{if } T \geqslant M_j, \end{cases} \quad (5)$$

where

$$\begin{aligned}\Pi_{1j}(n,p,T) &= \mathrm{TVP}_j(n,T) + \mathrm{TBP}_{1j}(p,T)\\ &= -\frac{1}{T}\left(\frac{S_V}{n} + S_B\right) + ap^{-\delta}\{p - c - F_j\\ &\quad + (pI_{Be} - vI_{Vp})M_j - \frac{T}{2}\{vr_B + pI_{Be}\\ &\quad + c(r_V + I_{Vp})[(n-1)(1-\rho)+\rho]\}\},\end{aligned} \quad (6)$$

and

$$\begin{aligned}\Pi_{2j}(n,p,T) &= \mathrm{TVP}_j(n,T) + \mathrm{TBP}_{2j}(p,T)\\ &= -\frac{1}{T}\left(\frac{S_V}{n} + S_B\right) + ap^{-\delta}\{p - c - F_j\\ &\quad + v(I_{Bp} - I_{Vp})M_j + \frac{(pI_{Be} - vI_{Bp})M_j^2}{2T}\\ &\quad - \frac{T}{2}\{v(r_B + I_{Bp}) + c(r_V + I_{Vp})[(n-1)(1-\rho)+\rho]\}\}.\end{aligned} \quad (7)$$

Our task here is to determine the buyer's optimal retail price, p^*, the optimal replenishment cycle length, T^*, and the optimal number of shipments per production run from the supplier to the buyer, n^*, which maximize the joint total profit per unit time, $\Pi_j(n,p,T)$, for all $j = 1, 2, \ldots, k$. Once the optimal solution (p^*, T^*) is arrived at the buyer's optimal order quantity per order $Q^* = D(p^*)T^* = a(p^*)^{-\delta}T^*$ follows.

4. Solution procedure

To examine the effect of n on the joint total profit per unit time, we first take the second-order partial derivative of (5) with respect to n, for $j = 1,2, \ldots ,k$, to obtain

$$\frac{\partial^2 \Pi_j(n,p,T)}{\partial n^2} = \frac{\partial^2 \Pi_{ij}(n,p,T)}{\partial n^2} = -\frac{2S_V}{n^3 T} < 0 \ \text{ for } i = 1,2.$$

The results identify $\Pi_j(n, p, T)$, for $j = 1, 2, \ldots, k$, as a concave function in n for fixed p and T. Thus, the search for the optimal shipment number, n^*, is reduced to finding a local optimal solution.

4.1. Determination of the optimal replenishment cycle length T for any given n and p

Having already decided n and p, we let T_j^* denote the optimal replenishment cycle length which maximizes $\Pi_j(n, p, T)$ in (5), for $j = 1, 2, \ldots, k$. Based on Eq. (6), by taking the second-order partial derivative of $\Pi_{1j}(n, p, T)$, $j = 1, 2, \ldots, k$, with respect to T for fixed n and p, we have

$$\frac{\partial^2 \Pi_{1j}(n,p,T)}{\partial T^2} = -\frac{2\bar{S}}{T^3} < 0, \quad \text{where } \bar{S} = \frac{S_V}{n} + S_B.$$

So for fixed n and p, $\Pi_{1j}(n, p, T)$ is a concave function in T. Thus, there exists a unique value of T (denoted by T_{1j}), which maximizes $\Pi_{1j}(n, p, T)$. T_{1j} can be obtained by solving the equation $\partial \Pi_{1j}(n, p, T)/\partial T = 0$, and is given by

$$T_{1j} = \sqrt{\frac{2\bar{S}}{ap^{-\delta}[(vr_B + pI_{Be}) + c\varphi]}}, \tag{8}$$

where $\varphi = (r_V + I_{Vp})[(n-1)(1-\rho)+\rho]$.

To ensure $T_{1j} < M_j$ (i.e., Case 1), we substitute (8) into inequality $T_{1j} < M_j$, and obtain

$$\text{if and only if } T_{1j} < M_j, \ \text{ then } 2\bar{S} < \Delta_j, \text{ where}$$
$$\Delta_j \equiv ap^{-\delta}(vr_B + pI_{Be} + c\varphi)M_j^2. \tag{9}$$

Substituting (8) into (6), we can obtain the joint total profit function for Case 1:

$$\begin{aligned}\Pi_{1j}(n,p) &\equiv \Pi_{1j}(n,p,T_{1j}) \\ &= ap^{-\delta}[p - c - F_j + (pI_{Be} - vI_{Vp})M_j] \\ &\quad - \sqrt{2ap^{-\delta}\bar{S}(vr_B + pI_{Be} + c\varphi)}.\end{aligned} \tag{10}$$

Next, for fixed n and p, solve the equation $\partial \Pi_{2j}(n, p, T)/\partial T = 0$, we obtain the value of T (denoted by T_{2j}) as

$$T_{2j} = \sqrt{\frac{2\bar{S} - ap^{-\delta}(pI_{Be} - vI_{Bp})M_j^2}{ap^{-\delta}[v(r_B + I_{Bp}) + c\varphi]}}. \tag{11}$$

Substituting (11) into (7), we can obtain the joint total profit function for Case 2:

$$\begin{aligned}\Pi_{2j}(n,p) &\equiv \Pi_{2j}(n,p,T_{2j}) \\ &= ap^{-\delta}[p - c - F_j + v(I_{Bp} - I_{Vp})M_j] \\ &\quad - \sqrt{ap^{-\delta}[v(r_B + I_{Bp}) + c\varphi]} \\ &\quad \sqrt{2\bar{S} - ap^{-\delta}(pI_{Be} - vI_{Bp})M_j^2}.\end{aligned} \tag{12}$$

To ensure $T_{2j} \geqslant M_j$ (i.e., Case 2), we substitute (11) into inequality $T_{2j} \geqslant M_j$, and obtain:

$$\text{if and only if } T_{2j} \geqslant M_j, \ \text{ then } 2\bar{S} \geqslant \Delta_j \text{ where}$$
$$\Delta_j \text{ is defined as in (9).} \tag{13}$$

Note that when $2\bar{S} \geqslant \Delta_j$, it can be shown that $2\bar{S} - ap^{-\delta}(pI_{Be} - vI_{Bp})M_j^2 > 0$ (for the proof see Appendix A), which implies T_{2j} in (11) is well defined. Furthermore, we have

$$\frac{\partial^2 \Pi_{2j}(n,p,T)}{\partial T^2} = -\frac{1}{T^3} \times [2\bar{S} - ap^{-\delta}(pI_{Be} - vI_{Bp})M_j^2] < 0$$

Thus, for fixed n and p, T_{2j} in (11) is a unique value which maximizes $\Pi_{2j}(n, p)$.

Combining (9) and (13), we obtain the following theorem.

Theorem 1. *For any given n, p and $j = 1, 2, \ldots, k$, we have*

(a) *If $2\bar{S} < \Delta_j$, then the buyer's optimal replenishment cycle length is $T_j^* = T_{1j}$.*
(b) *If $2\bar{S} \geqslant \Delta_j$, then the buyer's optimal replenishment cycle length is $T_j^* = T_{2j}$.*

Proof. It immediately follows from (9) and (13). □

It is easy to show that T_{ij}, $i = 1, 2$; $j = 1, 2, \ldots ,k$ has the following results, the proofs are omitted.

Property 1. $T_{11} = T_{12} = \cdots = T_{1k}$.

Property 2. $T_{21} > T_{22} > \cdots > T_{2k}$.

With $0 < M_1 < M_2 < \cdots < M_k$, $pI_{Be} > vI_{Bp}$, $I_{Bp} \geqslant I_{Vp}$ as well as (10) and (12), we can easily obtain the following property, the proof is omitted.

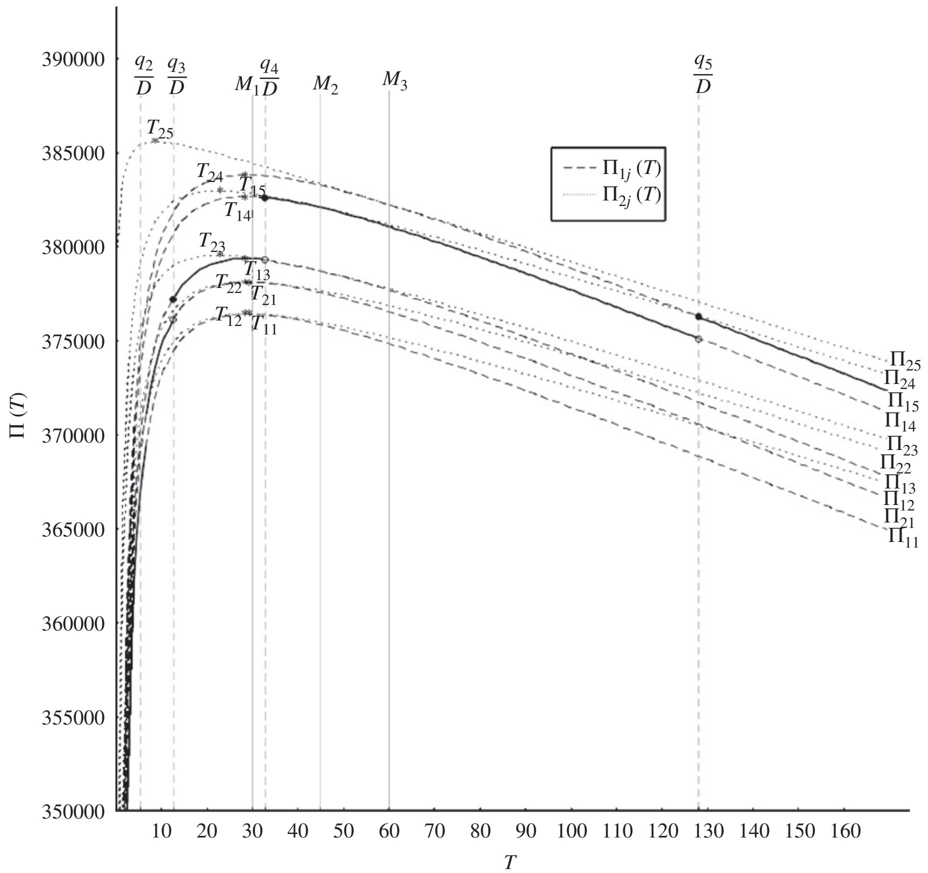
Property 3. $\Pi_{i1}(n,p,T_{i1}) < \Pi_{i2}(n,p,T_{i2}) < \cdots < \Pi_{ik}(n,p,T_{ik})$, for $i = 1,2$.

To facilitate the explanations of properties 1, 2 and 3, we present Fig. 2, which shows the shape of $\Pi_{ij}(n, p, T)$ with $n = 31$ and $p = 13.964$, while the other parameter values are given as in Example 1 for $i = 1, 2$; $j = 1, 2, 3, 4, 5$.

Now, we can obtain the following main result:

Theorem 2. *For any given n, p and $j = 1, 2, \ldots, k$, let $Q_j = D(p)T_j^*$, we have*

(a) *If $q_j \leqslant Q_j < q_{j+1}$, then $\Pi_j(n, p, T)$ at point $T = T_j^*$ has a maximum value.*
(b) *If $Q_j \geqslant q_{j+1}$, the lot size ordering by the buyer is over the upper-bound under the credit period M_j, then T_j^* is not a feasible solution.*
(c) *If $Q_j < q_j$, the lot size ordering by the buyer is less than the lower-bound under the credit period M_j, then T_j^* is not a feasible solution. However, $\Pi_j(n, p, T)$ is a strictly decreasing function in $T \in [q_j/D, q_{j+1}/D]$, hence, $\Pi_j(n, p, T)$ at point $T = q_j/D$ has a maximum value.*



Note: —— denotes the graph of Π_{ij} (T) on the region of feasible solutuions

Fig. 2. The relationship between inventory cycle and total profit of entire channel under quantity-dependent trade credit and freight rate (when $n = 31$ and $p = 13.964$).

Proof. It immediately follows from Theorem 1, $\Pi_j(n, p, T)$ is a concave function of T, and the discount schedule restructured from assumptions 8 and 13. □

Next, using the similar solution processes as Teng et al. (2005), we can find the optimal solution for p.

4.2. *Determination of the buyer's optimal retail price p for any given n*

Theorem 2 indicates that the buyer's optimal replenishment cycle length is either T_j^* (when $q_j \leqslant Q_j = D(p)T_j^* < q_{j+1}$) or q_j/D. In the remaining part of this section we will discuss the two situations in detail.

Situation 1. The buyer's optimal replenishment cycle length is T_j^* (when $q_j \leqslant Q_j = D(p)T_j^* < q_{j+1}$).

For fixed n, motivated by (9) and (13), we let

$$f_j(p) = \Delta_j = ap^{-\delta}(vr_{\mathrm{B}} + pI_{\mathrm{Be}} + c\varphi)M_j^2.$$

Because $c = c_0 + c_1/R + c_2R$ and $R = D/\rho$, the above equation can be rewritten as

$$f_j(p) = ap^{-\delta}\left\{vr_{\mathrm{B}} + pI_{\mathrm{Be}} + \left(c_0 + \frac{c_1\rho}{ap^{-\delta}} + \frac{c_2ap^{-\delta}}{\rho}\right)\varphi\right\}M_j^2.$$

Taking the derivative of $f_j(p)$ with respect to p, we have

$$\frac{\mathrm{d}f_j(p)}{\mathrm{d}p} = -ap^{-\delta-1}M_j^2\Big[(\delta-1)pI_{\mathrm{Be}} + 2ap^{-\delta}\frac{\delta}{\rho}c_2\varphi + \delta(c_0\varphi + vr_{\mathrm{B}})\Big] < 0. \quad (14)$$

So $f_j(p)$ is a strictly decreasing function of p. Furthermore, because $\lim_{p\to 0} f_j(p) = \infty$ and $\lim_{p\to\infty} f_j(p) = 0$, therefore, we can find a unique value p_{0j} such that $f_j(p_{0j}) = 2\bar{S}$, that is

$$2\bar{S} = ap_{0j}^{-\delta}(vr_{\mathrm{B}} + p_{0j}I_{\mathrm{Be}} + c\varphi)M_j^2. \quad (15)$$

Thus, (9) and (13) becomes

$$\text{if and only if } T_{1j} < M_j, \text{ then } p < p_{0j}, \quad (16)$$

and

$$\text{if and only if } T_{2j} \geqslant M_j, \text{ then } p \geqslant p_{0j}, \quad (17)$$

respectively, where p_{0j} is the value that satisfies (15).

Consequently, from (16) and (17), our task becomes that of finding the optimal value of p which maximizes the following joint total profit function when n is fixed:

$$\Pi_j(n,p) = \begin{cases} \Pi_{1j}(n,p) & \text{if } p < p_{0j}, \\ \Pi_{2j}(n,p) & \text{if } p \geqslant p_{0j}, \end{cases} \tag{18}$$

where $\Pi_{1j}(n, p)$ and $\Pi_{2j}(n, p)$ are given as in (10) and (12), respectively.

The optimal value of p which maximizes $\Pi_{1j}(n, p)$ can be determined by solving the first-order necessary condition (i.e., $\partial\Pi_{1j}(n, p)/\partial p = 0$) and examining the second-order condition for concavity (i.e., $\partial^2\Pi_{1j}(n, p)/\partial p^2 < 0$). Likewise, the optimal value of p which maximizes $\Pi_{2j}(n, p)$ can be determined by solving the first-order necessary condition (i.e., $\partial\Pi_{2j}(n, p)/\partial p = 0$) and examining the second-order condition for concavity (i.e., $\partial^2\Pi_{2j}(n, p)/\partial p^2 < 0$). The full derivations for $\partial\Pi_{1j}(n, p)/\partial p$, $\partial^2\Pi_{1j}(n, p)/\partial p^2$, $\partial\Pi_{2j}(n, p)/\partial p$ and $\partial^2\Pi_{2j}(n, p)/\partial p^2$ are given in Appendix B.

Situation 2. The buyer's optimal replenishment cycle length is q_j/D

In this situation, we substitute $T_{ij} = q_j/D = q_j/(ap^{-\delta})$, $i = 1, 2$, into (6) and (7), respectively, to obtain the joint total profit function:

$$\Pi_{1j}(n,p) \equiv \Pi_{1j}(n,p,T_{1j}) = -\frac{ap^{-\delta}\bar{S}}{q_j} + ap^{-\delta} \times \left\{ p - c - F_j + (pI_{\mathrm{Be}} - vI_{\mathrm{Vp}})M_j - \frac{q_j}{2ap^{-\delta}}(vr_{\mathrm{B}} + pI_{\mathrm{Be}} + c\varphi) \right\}, \tag{19}$$

and

$$\Pi_{2j}(n,p) \equiv \Pi_{2j}(n,p,T_{2j}) = -\frac{ap^{-\delta}\bar{S}}{q_j} + ap^{-\delta} \times \left\{ p - c - F_j + v(I_{\mathrm{Bp}} - I_{\mathrm{Vp}})M_j + \frac{(pI_{\mathrm{Be}} - vI_{\mathrm{Bp}})ap^{-\delta}M_j^2}{2q_j} - \frac{q_j}{2(ap^{-\delta})}[v(r_{\mathrm{B}} + I_{\mathrm{Bp}}) + c\varphi] \right\}. \tag{20}$$

Furthermore, it can be shown that

if and only if $T_{1j} < M_j$, then $p < \tilde{p}_{0j}$,

and

if and only if $T_{2j} \geqslant M_j$, then $p \geqslant \tilde{p}_{0j}$,

$$\text{where } \tilde{p}_{0j} = (aM_j/q_j)^{1/\delta}. \tag{21}$$

As a result, for Situation 2, our problem becomes that of finding the optimal value of p which maximizes the following joint total profit function when n is fixed:

$$\Pi_j(n,p) = \begin{cases} \Pi_{1j}(n,p) & \text{if } p < \tilde{p}_{0j}, \\ \Pi_{2j}(n,p) & \text{if } p \geqslant \tilde{p}_{0j}, \end{cases} \tag{22}$$

where $\Pi_{1j}(n, p)$ and $\Pi_{2j}(n, p)$ are given as in (19) and (20), respectively.

The optimal value of p which maximizes $\Pi_{1j}(n, p)$ can be determined by solving the first-order necessary condition (i.e., $\partial\Pi_{1j}(n, p)/\partial p = 0$) and examining the second-order condition for concavity (i.e., $\partial^2\Pi_{1j}(n, p)/\partial p^2 < 0$). Likewise, the optimal value of p which maximizes $\Pi_{2j}(n, p)$ can be determined by solving the first-order necessary condition (i.e., $\partial\Pi_{2j}(n, p)/\partial p = 0$) and examining the second-order condition for concavity (i.e., $\partial^2\Pi_{2j}(n, p)/\partial p^2 < 0$). The full derivations for $\partial\Pi_{1j}(n, p)/\partial p$, $\partial^2\Pi_{1j}(n, p)/\partial p^2$, $\partial\Pi_{2j}(n, p)/\partial p$ and $\partial^2\Pi_{2j}(n, p)/\partial p^2$ are given in Appendix C.

From the above arguments and Theorem 2, we can establish the following algorithm to obtain the optimal solution (n^*, p^*, T^*).

Algorithm 1.

Step 1 Set $n = 1$

Step 2 For $j = 1, 2, \ldots, k$, determine p_{0j} by solving (15)

- (a) If there exists a p_{1j} such that $p_{1j} < p_{0j}$ and p_{1j} satisfies $\partial\Pi_{1j}(n, p)/\partial p = 0$ and $\partial^2\Pi_{1j}(n, p)/\partial p^2 < 0$, where $\Pi_{1j}(n, p)$ is given as in (10), then determine $T_j^* = T_{1j}(p_{1j})$ from (8)
- (b) If there exists a p_{2j} such that $p_{2j} \geqslant p_{0j}$ and p_{2j} satisfies $\partial\Pi_{2j}(n, p)/\partial p = 0$ and $\partial^2\Pi_{2j}(n, p)/\partial p^2 < 0$, where $\Pi_{2j}(n, p)$ is given as in (12), then we determine $T_j^* = T_{2j}(p_{2j})$ from (11)

Step 3 Calculate $Q_j = DT_j^*$ and check each Q_j under M_j, $j = 1, 2, \ldots, k$

Step 3.1 For $j = 1, 2, \ldots, k-1$,

- (a) if $q_j \leqslant Q_j < q_{j+1}$, then T_j^* is a feasible solution, using (10) or (12) to obtain
  $$\Pi_j(n,p_j) = \begin{cases} \Pi_{1j}(n,p_{1j}) & \text{if } p_{1j} < p_{0j}, \\ \Pi_{2j}(n,p_{2j}) & \text{if } p_{2j} \geqslant p_{0j}. \end{cases}$$
- (b) if $Q_j \geqslant q_{j+1}$, then T_j^* is not a feasible solution. Set $\Pi_j(n, p_j) = 0$
- (c) if $Q_j < q_j$, then determine $\tilde{p}_{0j}$ by solving (21)
  - (i) If there exists a p_{1j} such that $p_{1j} < \tilde{p}_{0j}$ and p_{1j} satisfies $\partial\Pi_{1j}(n, p)/\partial p = 0$ and $\partial^2\Pi_{1j}(n, p)/\partial p^2 < 0$, where $\Pi_{1j}(n, p)$ is given as in (19), then set $T_j^* = T_{1j}(p_{1j}) = q_j/(ap_{1j}^{-\delta})$. We use (19) to obtain $\Pi_j(n, p_j) = \Pi_{1j}(n, p_{1j})$
  - (ii) If there exists a p_{2j} such that $p_{2j} \geqslant \tilde{p}_{0j}$ and p_{2j} satisfies $\partial\Pi_{2j}(n, p)/\partial p = 0$ and $\partial^2\Pi_{2j}(n, p)/\partial p^2 < 0$, where $\Pi_{2j}(n, p)$ is given as in (20), then set $T_j^* = T_{2j}(p_{2j}) = q_j/(ap_{2j}^{-\delta})$. We use (20) to obtain $\Pi_j(n, p_j) = \Pi_{2j}(n, p_{2j})$

Step 3.2 For $j = k$

- (a) if $Q_k \geqslant q_k$, then T_k^* is a feasible solution, using (10) or (12) to obtain
  $$\Pi_k(n,p_k) = \begin{cases} \Pi_{1k}(n,p_{1k}) & \text{if } p_{1k} < p_{0k}, \\ \Pi_{2k}(n,p_{2k}) & \text{if } p_{2k} \geqslant p_{0k}. \end{cases}$$
- (b) if $Q_k < q_k$, then determine $\tilde{p}_{0k}$ by solving (21)
  - (i) If there exists a p_{1k} such that $p_{1k} < \tilde{p}_{0k}$ and p_{1k} satisfies $\partial\Pi_{1j}(n, p)/\partial p = 0$ and $\partial^2\Pi_{1j}(n, p)/\partial p^2 < 0$, where $\Pi_{1j}(n, p)$ is given as in (19), then set $T_k^* = T_{1k}(p_{1k}) = q_k/(ap_{1k}^{-\delta})$. We use (19) to obtain $\Pi_k(n, p_k) = \Pi_{1k}(n, p_{1k})$
  - (ii) If there exists a p_{2k} such that $p_2 \geqslant \tilde{p}_{0k}$ and p_{2k} satisfies $\partial\Pi_{2j}(n, p)/\partial p = 0$ and $\partial^2\Pi_{2j}(n, p)/\partial p^2 < 0$, where $\Pi_{2j}(n, p)$ is given as in (20), then set $T_k^* = T_{2k}(p_{2k}) = q_k/(ap_{2k}^{-\delta})$. We use (20) to obtain $\Pi_k(n, p_k) = \Pi_{2k}(n, p_{2k})$

Step 4 Find $\max_{j=1,2,\ldots,k} \Pi_j(n,p_j)$. Set $\Pi^*(n,p^{(n)}) = \max_{j=1,2,\ldots,k} \Pi_j(n,p_j)$, then, for given n, $p^{(n)}$ is the optimal retail price and the corresponding replenishment cycle length $T^{(n)}$ is the optimal replenishment cycle length

Step 5 Let $n = n+1$ repeats Steps 2–4 to find $\Pi^*(n,p^{(n)})$

Step 6 If $\Pi^*(n,p^{(n)}) \geqslant \Pi^*(n-1,p^{(n-1)})$, go to Step 5. Otherwise the optimal solution is $(n^*,p^*,T^*) = (n-1,p^{(n-1)},T^{(n-1)})$

5. Numerical examples

Example 1. In order to illustrate the above solution procedure, let us consider an inventory system with the data: $a = 10^6$, $\delta = 1.5$, $\rho = 0.95$, $S_V = 1000$, $S_B = 200$, $r_V = 0.05$, $r_B = 0.1$, $I_{Vp} = 0.04$, $I_{Be} = 0.09$, $I_{Bp} = 0.10$, $c_0 = 1$, $c_1 = 2.5 \times 10^4$, $c_2 = 2.5 \times 10^{-5}$, $v = 7$ and the product weighs $\theta = 2$ lbs per unit. In addition, we assume that the supplier offers credit terms as follows:

η	Q (units/order)	N_η (days)
1	$0 \leqslant Q < 1000$	30
2	$1000 \leqslant Q < 10{,}000$	45
3	$10{,}000 \leqslant Q$	60

A freight rate schedule is offered below:

ε	W (lbs/ship)	ψ_ε (\$/lb)
1	$0 \leqslant W < 1000$	0.60
2	$1000 \leqslant W < 5000$	0.57
3	$5000 \leqslant W$	0.51

Combining the two tables, we obtain the following schedule

J	Q (units/order)	M (days)	F (\$/unit)
1	$0 \leqslant Q < 500$	30	1.20
2	$500 \leqslant Q < 1000$	30	1.14
3	$1000 \leqslant Q < 2500$	45	1.14
4	$2500 \leqslant Q < 10{,}000$	45	1.02
5	$10{,}000 \leqslant Q$	60	1.02

The optimal solutions obtained through Algorithm 1 are presented in Table 1 as System I. Optimal solutions from other policies are also presented in Table 1 as Systems II, III, IV, V and VI, to help illustrate the effects of the strategy represented by System I where both the freight rate and trade credit are linked to the order quantity.

Looking at the optimal decisions in System I, where the freight rate and credit period offered are dependent on the order quantity, the optimal annual joint total profit for the supply chain is \$382,614 and the optimal order quantity is 2500 units/order. Therefore, the buyer pays for the shipments at \$1.02/unit and pays the purchase off within 45-day after delivery. In addition, the optimal retail price is \$13.964/unit and the optimal replenishment cycle is 32.06 days. In this case, the supplier's production lot size is 31 times the order quantity or 77,500 units per setup.

The only difference between System II and System I is that System II assumes the credit period offered by the supplier is 30 days (i.e., "net30"). In Table 1, we show that the annual joint total profit is less in System II compared to System I. This result suggests that managers should consider a trade credit policy as a marketing strategy and link it with the order quantities to improve the supply chain performance. In System III, a quantity discount in the freight rate is offered without a corresponding trade credit offer. From Table 1, we show that under this condition, the buyer will order 2653 units per order to take advantage of the freight rate discount. However, a bigger lot size leads to a longer inventory cycle and fewer deliveries (lower value of n^*). To improve market demand the buyer reduces the retail price which in turn decreases the total profit for the entire supply chain compared to System I. We can also note that the channel's optimal joint total profits in Systems IV, V and VI (all with a fixed freight rate) are less than that in Systems I, II and III, where freight rate discounts are offered. With the results shown in Table 1, we can see a consistent positive impact on profits for the entire supply chain system when freight rate discounts are offered.

When comparing the optimal solutions between System I and System IV from the numbers in Table 1, we can see that, without the offer of freight rate discounts, the buyer will order less per order with a shorter replenishment cycle in System IV. This result illustrates that in such a situation the buyer shortens the replenishment cycle to take advantage of the trade credit more frequently. Furthermore, a higher retail price is set which results in a decrease in both demand and in the supplier's production size (i.e. 63,547 units/setup). Therefore, the total profit for the entire supply chain is less when compared to System I. Similarly, we see that the same thing happens when comparing System IV and System V. System IV outperforms System V, thus ruling out the possibility that System V can fare better than System I. In System VI, where neither trade credit nor freight rate discount is offered, just as expected, the entire supply chain has the lowest profits compared to the profit results from the other systems. With System I being the superior performer compared to the alternatives, it is clear that the whole supply chain system benefits from a trade policy that links both the trade credit and freight rate discount to the order quantity.

Example 2. In this example, we study the effects of variable capacity utilization ρ. Consider different $\rho \in \{0.15, 0.25, 0.50, 0.75, 0.95\}$, keeping the values of other parameters the same as in Example 1. The algorithm is applied to obtain the optimal solutions shown in Table 2.

It can be seen from Table 2 that as ρ increases, the retail price is substantially lower, which results in a substantial increase in demand. It is interesting that both the supplier's production size (n^*Q^*) and the annual profit for the entire supply chain increases with the increases in ρ. As a result, the closer the production rate is to the demand rate, the greater the gain is in the integrated mode. This implies that if the supplier and buyer would work in a cooperative manner to synchronize supply with actual customer demand, the channel's profit will improve.

Example 3. A sensitivity analysis is performed to test the robustness of the model when two given parameters, S_V and S_B, are changed. Table 3 illustrates the effects with

Table 1
Computation results for Example 1

System	Trade credit	Freight rate discounts	Optimal paying time	Optimal freight rate	n^*	Q^*	n^*Q^*	p^*	$D(p^*)$	$c(R^*)$	T^*	Profit ($)		
												Buyer	Supplier	System
I	Order-quantity related	Yes	45 days after delivery	1.02	31	2500	77,500	13.964	28,459	2.5835	32.06	168,860	213,754	382,614
II	"net 30"	Yes	30 days after delivery	1.02	31	2500	77,500	13.969	28,446	2.5835	32.08	167,449	213,986	381,436
III	No	Yes	Cash on delivery	1.02	27	2653	71,635	13.930	28,554	2.5832	33.91	164,742	215,440	380,182
IV	Order-quantity related	No	45 days after delivery	1.20 (fixed)	28	2270	63,547	14.300	27,561	2.5870	30.06	167,812	207,113	374,926
V	"net 30"	No	30 days after delivery	1.20 (fixed)	28	2270	63,563	14.306	27,546	2.6096	30.08	166,421	207,321	373,742
VI	No	No	Cash on delivery	1.20 (fixed)	27	2596	70,083	14.384	27,345	2.5881	34.65	165,168	206,427	371,594

$\pm 50\%$ variations of the given parameters. In order to facilitate a clear comparison, we use the same numbers as are used in Example 1 except for the two noted parameters in Table 3.

When the setup cost S_V is higher, the supplier tends to produce more in one production run, thus forcing the system to experience more frequent deliveries. In such a case, the buyer increases the retail price, which in turn leads to a decrease in product demand and the channel's profit suffers as a result. If the buyer's ordering cost S_B is adjusted upward, the buyer will increase the optimal order quantity to 10,000 units per order. At this lot size the buyer can benefit from the supplier's offer of a 60-day credit period and enjoys a low $1.02 per unit freight rate.However, such a large lot size lengthens the optimal inventory cycle to 127.18 days. To improve product demand the buyer reduces the retail price, and, as would be expected, the joint total profit $\Pi(n, p, T)$ is less than what it is in the base case.

6. Summary and conclusions

In this paper, we first formulated an integrated supplier–buyer inventory model with a few assumptions. The market demand is sensitive to the retail price and the order size is linked to both the trade credit and freight rate. The production rate will react to the market demand rate. Furthermore, the production cost is a convex function of the production rate. By analyzing the total channel profit function as explained in Section 4, we then developed a solution algorithm to simultaneously determine the optimal order quantity, retail price and the number of shipments per production run from the supplier to the buyer.

This paper demonstrates that significant profit increase potential occurs for the entire supply chain by linking the trade credit policy and freight rate policy to the order quantity. The results in the numerical examples indicate that the ratio between the buyer's demand rate and the supplier's production rate is an important parameter for the channel's profit. When the production rate is close to the demand rate, the supplier and buyer can work in a cooperative manner, synchronizing supply with actual customer demand, to maximize the channel's profit.

In reality, industries have unique needs in transportation logistics management to satisfy the delivery and service requirements of their customers. These needs account for the wide range of shipping options and freight rates. The shipment could be local, domestic or international. The load could be LTL, TL or small package. Or the route the freight is transported on can be land, sea, air or any combination of the three. These are all examples of freight options. Moreover, increasing emphasis on "the perfect order" is driving many companies to focus on product damage reduction in shipping. It is only logical that more transportation policy considerations need to be part of future researches. Another extension of this work may be set in the direction of considering demand as a function of price and time. Similarly, we can extend this model to allow for shortages and backorders. Finally, while this paper deals with a situation that allows only for

Table 2
Computation results for Example 2

ρ	Optimal paying time	Optimal freight rate	n^*	Q^*	n^*Q^*	p^*	$D(p^*)$	$c(R^*)$	T^*	$\Pi(n, p, T)$
0.15	45 days after delivery	1.02	4	2500	10,000	25.122	12,880	3.4379	70.84	325,303
0.25	45 days after delivery	1.02	6	2500	15,000	21.070	16,332	3.0159	55.87	340,372
0.50	45 days after delivery	1.02	8	2500	20,000	16.873	22,044	2.6693	41.39	360,743
0.75	45 days after delivery	1.02	13	2500	32,500	14.956	25,941	2.5875	35.18	373,651
0.95	45 days after delivery	1.02	31	2500	77,500	13.964	28,459	2.5835	32.06	382,614

Table 3
Computation results for Example 3

			Optimal paying time	Optimal freight rate	n^*	Q^*	n^*Q^*	p^*	$D(p^*)$	$c(R^*)$	T^*	$\Pi(n, p, T)$
The base case			45 days after delivery	1.02	31	2500	77,500	13.964	28,459	2.5835	32.06	382,614
Parameter change	S_V	+50%	45 days after delivery	1.02	37	2500	92,500	13.977	28,424	2.5836	32.10	382,325
		−50%	45 days after delivery	1.02	22	2500	55,000	13.948	28,504	2.5833	32.01	382,991
	S_B	+50%	60 days after delivery	1.02	8	10,000	80,000	13.878	28,698	2.5828	127.18	378,592
		−50%	45 days after delivery	1.02	31	2500	77,500	13.866	28,732	2.5827	31.76	384,595

equal shipments, this assumption may be relaxed so unequal shipments can be looked at as well.

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Appendix A. To show that $2\bar{S} - ap^{-\delta}(pI_{Be} - vI_{Bp})M_j^2 > 0$, if $2\bar{S} \geqslant \Delta_j$

Because $\Delta_j = ap^{-\delta}(vr_B + pI_{Be} + c\varphi)M_j^2$, thus, from $2\bar{S} \geqslant \Delta_j$ we have

$$2\bar{S} \geqslant ap^{-\delta}(vr_B + pI_{Be} + c\varphi)M_j^2. \tag{A.1}$$

Therefore, it becomes

$$\begin{aligned}&2\bar{S} - ap^{-\delta}(pI_{Be} - vI_{Bp})M_j^2 \geqslant ap^{-\delta}(vr_B + pI_{Be} + c\varphi)M_j^2\\ &- ap^{-\delta}(pI_{Be} - vI_{Bp})M_j^2 = ap^{-\delta}[v(r_B + I_{Bp}) + c\varphi]M_j^2 > 0.\end{aligned} \tag{A.2}$$

The proof is completed.

Appendix B

The full derivations for $\partial\Pi_{1j}(n, p)/\partial p$, $\partial^2\Pi_{1j}(n, p)/\partial p^2$, $\partial\Pi_{2j}(n, p)/\partial p$ and $\partial^2\Pi_{2j}(n, p)/\partial p^2$, where $\Pi_{1j}(n, p)$ and $\Pi_{2j}(n, p)$ are given as in Eqs. (10) and (12), respectively:

$$\begin{aligned}\frac{\partial \Pi_{1j}(n,p)}{\partial p} &= ap^{-\delta-1}\Big[\delta(c_0 + F_j + M_j v I_{Vp}) + 2\frac{\delta}{\rho}ap^{-\delta}c_2\\ &-(\delta-1)p(1+M_j I_{Be})\Big]\\ &+\frac{ap^{-\delta-1}\{\rho[\delta(c_0\varphi + vr_B) + (\delta-1)pI_{Be}] + 2\delta ap^{-\delta}c_2\varphi\}\sqrt{\bar{S}}}{\sqrt{2\rho\{ap^{-\delta}[\rho(vr_B + pI_{Be} + c_0\varphi) + ap^{-\delta}c_2\varphi] + \rho^2 c_1\varphi\}}} = 0,\end{aligned} \tag{B.1}$$

$$\begin{aligned}\frac{\partial^2 \Pi_{1j}(n,p)}{\partial p^2} &= -ap^{-\delta-2}\Big[(\delta+1)(c_0 + F_j + M_j v I_{Vp})\\ &+\frac{2\delta(2\delta+1)}{\rho}ap^{-\delta}c_2 + \delta(\delta-1)p(1+M_j I_{Be})\Big]\\ &+\sqrt{\frac{\bar{S}}{2\rho}}\,\frac{\{ap^{-\delta-1}\rho[\delta(vr_B + c_0\varphi) + (\delta-1)pI_{Be}] + 2\delta ap^{-\delta}c_2\varphi\}^2}{\{2ap^{-\delta}[\rho(vr_B + pI_{Be} + c_0\varphi) + ap^{-\delta}c_2\varphi + \rho^2 c_1\varphi/ap^{-\delta}]\}^{3/2}}\\ &- ap^{-\delta-2}\delta\{\rho[(\delta+1)(vr_B + c_0\varphi) + (\delta-1)pI_{Be}]\\ &+ 2(2\delta+1)ap^{-\delta}c_2\varphi\}\\ &\times\sqrt{\frac{\bar{S}}{2\rho\{ap^{-\delta}[\rho(vr_B + pI_{Be} + c_0\varphi) + ap^{-\delta}c_2\varphi] + \rho^2 c_1\varphi\}}},\end{aligned} \tag{B.2}$$

$$\begin{aligned}\frac{\partial \Pi_{2j}(n,p)}{\partial p} &= ap^{-\delta}\Big\{1 - \frac{\delta}{p}[p - c_0 - F_j + M_j v(I_{Bp} - I_{Vp})]\\ &+\frac{2\delta ap^{-\delta}c_2}{\rho p}\Big\} + \frac{\delta ap^{-\delta}}{2\rho p}\\ &\times\{\rho[v(r_B + I_{Bp}) + c_0\varphi] + 2ap^{-\delta}c_2\varphi\}\\ &\times\sqrt{\frac{\rho[2\bar{S} + ap^{-\delta}(vI_{Bp} - pI_{Be})M_j^2]}{ap^{-\delta}\rho[v(r_B + I_{Bp}) + c_0\varphi] + \rho^2 c_1\varphi + (ap^{-\delta})^2 c_2\varphi}}\\ &-\frac{ap^{-\delta}\left[(\delta-1)pI_{Be} - \delta vI_{Bp}\right]M_j^2}{2p}\\ &\times\sqrt{\frac{ap^{-\delta}\rho[v(r_B + I_{Bp}) + c_0\varphi] + \rho^2 c_1\varphi + (ap^{-\delta})^2 c_2\varphi}{\rho[2\bar{S} + ap^{-\delta}(vI_{Bp} - pI_{Be})M_j^2]}},\end{aligned} \tag{B.3}$$

$$\frac{\partial^2 \Pi_{2j}(n,p)}{\partial p^2} = -\delta a p^{-\delta-2}\{(\delta+1)[c_0 + F_j - M_j v(I_{\mathrm{Bp}} - I_{\mathrm{Vp}}) - p] + 2p + 2(2\delta+1)ap^{-\delta}\frac{c_2}{\rho}\}$$
$$- \frac{\delta(ap^{-\delta-1})^2[\delta v I_{\mathrm{Bp}} - (\delta-1)pI_{\mathrm{Be}}]\{\rho[v(r_{\mathrm{B}} + I_{\mathrm{Bp}}) + c_0\varphi] + 2ap^{-\delta}c_2\varphi\}M_j^2}{\sqrt{\rho[2\bar{S} + ap^{-\delta}(vI_{\mathrm{Bp}} - pI_{\mathrm{Be}})M_j^2]}\sqrt{ap^{-\delta}\rho[v(r_{\mathrm{B}} + I_{\mathrm{Bp}}) + c_0\varphi] + \rho^2 c_1\varphi + (ap^{-\delta})^2 c_2\varphi}}$$
$$+ \left\{\frac{\delta a p^{-\delta-2}[(\delta+1)vI_{\mathrm{Bp}} - (\delta-1)pI_{\mathrm{Be}}]M_j^2}{2} + \frac{(ap^{-\delta-2}M_j^2)^2[\delta v I_{\mathrm{Bp}} - (\delta-1)pI_{\mathrm{Be}}]^2}{4[2\bar{S} + ap^{-\delta}(vI_{\mathrm{Bp}} - pI_{\mathrm{Be}})M_j^2]}\right\}$$
$$\times \sqrt{\frac{\rho\left[2\bar{S} + ap^{-\delta}(vI_{\mathrm{Bp}} - pI_{\mathrm{Be}})M_j^2\right]}{ap^{-\delta}\rho[v(r_{\mathrm{B}} + I_{\mathrm{Bp}}) + c_0\varphi] + \rho^2 c_1\varphi + (ap^{-\delta})^2 c_2\varphi}}$$
$$- ap^{-\delta-2}\left\{\frac{(\delta+1)}{2}[v(r_{\mathrm{B}} + I_{\mathrm{Bp}}) + c_0\varphi] + ap^{-\delta}(2\delta+1)\frac{c_2\varphi}{\rho}\right\}$$
$$\times \sqrt{\frac{ap^{-\delta}\rho[v(r_{\mathrm{B}} + I_{\mathrm{Bp}}) + c_0\varphi] + \rho^2 c_1\varphi + (ap^{-\delta})^2 c_2\varphi}{\rho[2\bar{S} + ap^{-\delta}(vI_{\mathrm{Bp}} - pI_{\mathrm{Be}})M_j^2]}}$$
$$+ \sqrt{ap^{-\delta}\rho[v(r_{\mathrm{B}} + I_{\mathrm{Bp}}) + c_0\varphi] + \rho^2 c_1\varphi + (ap^{-\delta})^2 c_2\varphi} \times \sqrt{\rho[2\bar{S} + ap^{-\delta}(vI_{\mathrm{Bp}} - pI_{\mathrm{Be}})M_j^2]}$$
$$\times \left\{\frac{\delta a p^{-\delta}\{\rho[v(r_{\mathrm{B}} + I_{\mathrm{Bp}}) + c_0\varphi] + 2ap^{-\delta}c_2\varphi\}}{2p\{ap^{-\delta}\{\rho[v(r_{\mathrm{B}} + I_{\mathrm{Bp}}) + c_0\varphi] + ap^{-\delta}c_2\varphi\} + \rho^2 c_1\varphi\}}\right\}^2. \tag{B.4}$$

Appendix C

The full derivations for $\partial\Pi_{1j}(n,\ p)/\partial p$, $\partial^2\Pi_{1j}(n,\ p)/\partial p^2$, $\partial\Pi_{2j}(n,\ p)/\partial p$ and $\partial^2\Pi_{2j}(n,\ p)/\partial p^2$ where $\Pi_{1j}(n,\ p)$ and $\Pi_{2j}(n,\ p)$ are given as in Eqs. (19) and (20), respectively:

$$\frac{\partial \Pi_{1j}(n,p)}{\partial p} = \frac{ap^{-\delta-1}}{2}\left[2\delta(c_0 + F_j + vM_j I_{\mathrm{Vp}}) - 2(\delta-1) \times p(1 + M_j I_{\mathrm{Be}}) + \frac{\delta c_2\varphi q_j}{\rho} + \frac{2\delta\bar{S}}{q_j}\right] + \frac{2\delta c_2}{\rho p}(ap^{-\delta})^2 - \frac{\delta\rho c_1\varphi q_j}{2ap^{-\delta-1}} - \frac{q_j}{2}I_{\mathrm{Be}}, \tag{C.1}$$

$$\frac{\partial^2 \Pi_{1j}(n,p)}{\partial p^2} = -2\frac{\delta(2\delta+1)}{\rho}(ap^{-\delta-1})^2 c_2 - \frac{\delta(\delta-1)\rho c_1\varphi}{2ap^{-\delta+2}}q_j + ap^{-\delta-2}\left\{\delta(\delta-1)p(1 + M_j I_{\mathrm{Be}}) -(\delta+1)\left[\delta(c_0 + F_j + vM_j I_{\mathrm{Vp}}) + \frac{\delta}{2\rho}c_2\varphi q_j + \delta(\bar{S}/q_j)\right]\right\}, \tag{C.2}$$

$$\frac{\partial \Pi_{2j}(n,p)}{\partial p} = ap^{-\delta-1}\left\{\delta\left[c_0 + F_j - vM_j(I_{\mathrm{Bp}} - I_{\mathrm{Vp}}) - \frac{c_2\varphi}{2\rho}q_j + \frac{\bar{S}}{q_j}\right] - (\delta-1)p\right\} + (ap^{-\delta})^2 \times \left\{\frac{1}{q_j}\left[(1/2 - \delta)I_{\mathrm{Be}} + (\delta/p)vI_{\mathrm{Bp}}\right]M_j^2 - \frac{2\delta c_2}{\rho p}\right\}, \tag{C.3}$$

$$\frac{\partial^2 \Pi_{2j}(n,p)}{\partial p^2} = -\delta a p^{-\delta-2}\left\{\frac{(\delta+1)}{2}\left[c_0 + F_j - vM_j(I_{\mathrm{Bp}} - I_{\mathrm{Vp}}) - \frac{c_2\varphi}{2\rho}q_j + \frac{2\bar{S}}{q_j}\right] - (\delta-1)p\right\} - \delta(ap^{-\delta-1})^2\left\{\frac{1}{q_j}[(1-2\delta)pI_{\mathrm{Be}} +(1+2\delta)vI_{\mathrm{Bp}}]M_j^2 + \frac{2(1+2\delta)c_2}{\rho}\right\} - \frac{\delta(\delta-1)\rho c_1}{2ap^{-\delta+2}}q_j. \tag{C.4}$$

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